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Quantization on nilpotent Lie groups. (English) Zbl 1347.22001

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The book is devoted to pseudo-differential calculus in the setting of a graded nilpotent Lie group G. Pseudo-differential operators on compact Lie groups were already considered by the second author and {\it V. Turunen} [Pseudo-differential operators and symmetries. Background analysis and advanced topics. Pseudo-Differential Operators. Theory and Applications 2. Basel: Birkhäuser, xiv, 709 p. (2010; Zbl 1193.35261)]. Symbols were defined there as matrix-valued functions, thanks to the fact that the unitary irreducible representations are all finite-dimensional in that case. The situation in the present case is much more involved, since the symbol $\sigma(x,\pi)$, with $x \in G$, $\pi \in \widehat{G}$, is now a family of operators on the corresponding Hilbert space H_π. The standard quantization $T = 0p(\sigma)$, given by

$$Tu(x) = \int_{\widehat{G}} \mathrm{Tr}(\pi(x)\sigma(x,\pi)\widehat{u}(\pi))\, d\mu(\pi),$$

is then problematic, in particular when the operators of the family are unbounded. As related contribution let us mention {\it M. E. Taylor} [Noncommutative harmonic analysis. Mathematical Surveys and Monographs, 22. Providence, RI: American Mathematical Society (AMS). XVI, 328 p. (1986; Zbl 0604.43001)]. In the present book, the authors are able to give a precise meaning to the quantization formula by using the theory of Rockland operators and their associated Sobolev spaces. The content of chapters is precisely the following: Chapter 1 provides the reader with the basic preliminary facts about Lie groups. In Chapter 2 the authors briefly review the contents of the above-mentioned book of M. Ruzhansky and V. Turunen [loc. cit.], and subsequent developments for compact groups. Chapter 3 is devoted to the theory of homogeneous Lie groups, with emphasis on graded Lie groups. As a preparation to the quantization proceeding, Rockland oprators are studied in detail in Chapter 4. The core of the book is Chapter 5, where quantization is defined, and symbolic calculus is presented. Chapter 6 treats the particular case of pseudo-differential operators on the Heisenberg group. To the benefit of non-expert readers, group C^* and von Neumann algebras are reviewed in a final appendix. In conclusion, we want to remark that the contents of the volume are extremely rich. Beside presenting the new theory in the graded nilpotent case, the authors offer a complete view of the calculus of pseudo-differential operators on groups giving detailed references to preceding contributions. Also, we note the big effort to provide a self-contained presentation, addressed to a large audience. This monograph was the winner of the 2016 Ferran Sunyer i Balanguer prize. Reviewer: Luigi Rodino (Torino)

MSC:

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22E25	Nilpotent and solvable Lie groups
22C05	Compact topological groups
35S99	Pseudodifferential operators

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