

Zbl pre05500958

Ruzhansky, Michael; Turunen, Ville**Pseudo-differential operators and symmetries. Background analysis and advanced topics.** (English)

Pseudo-Differential Operators 2. Basel: Birkhäuser. xiv, 709 p. EUR 69.95/net; SFR 109.00; \$ 79.95; £ 62.99 (2010). ISBN 978-3-7643-8513-2/pbk; ISBN 978-3-7643-8514-9/ebook

<http://dx.doi.org/10.1007/978-3-7643-8514-9>

The book is devoted to a global theory of pseudo-differential operators on compact Lie groups. Peculiarity of the presentation, with respect to standard specialistic monographies, is that the authors devote a large portion of the work, namely Part I, Foundations of Analysis, and Part III, Representation Theory of Compact Groups, about 300 pages altogether, to provide background material surrounding the theory. Beside giving self-contained exposition, these parts may be used for independent learning of topics as Measure Theory and Integration, Topological Groups, etc..

The core of the book, concerning research results for the first time in textbook form, is in Part II, Commutative Symmetries, and Part IV, Noncommutative Symmetries. Namely, in Part II, after reviewing Fourier Analysis in \mathbb{R}^n and classical pseudo-differential calculus, the authors treat pseudo-differential operators on the n -torus \mathbb{T}^n . The main ideas here come from a celebrated paper of *M. S. Agranovich* [Funkts. Anal. Prilozh. 13, No. 4, 54–56 (1979; Zbl 0423.35087) and English translation in Funct. Anal. Appl. 13, 279–281 (1980; Zbl 0437.35073)] and from the Ph.D. Thesis of *V. Turunen*, Helsinki University of Technology (2001), and are based on toroidal symbols $a(x, \eta)$, $x \in \mathbb{T}^n$, $\eta \in \mathbb{Z}^n$, satisfying the estimates

$$|\Delta_\eta^\alpha \partial_x^\beta a(x, \eta)| \leq C_{\alpha\beta} (1 + |\eta|)^{m - \rho|\alpha| + \delta|\beta|},$$

where Δ_η^α are differences operators. The corresponding pseudo-differential operators, expressed by Fourier series, coincide with the locally defined $S_{\rho, \delta}^m$ -operators on the manifold \mathbb{T}^n .

The advantage of the use of the toroidal symbols is that they allow a global calculus, with nice applications to partial differential operators on \mathbb{T}^n .

In Part IV the author introduce a noncommutative quantization of pseudo-differential operators on a compact Lie group G . The full symbol $a(x, \eta)$ is a mapping defined globally on $G \times \widehat{G}$, where \widehat{G} is the unitary dual of G . The main difference with the toroidal quantization is that, due to the noncommutativity of the group, symbols become matrix-valued with sizes depending on the dimension of the unitary irreducible representations of the group, which are finite-dimensional because of the assumption of compactness (on the n -torus they are one-dimensional).

The authors note that for every closed simply-connected three-dimensional manifold M , by the recently solved Poincaré conjecture, there is a global diffeomorphism $M \cong S^3 \cong \text{SU}(2)$ that turns M into the Lie group induced by $\text{SU}(2)$. Particular attention is then devoted to pseudo-differential operators on $\text{SU}(2)$. Homogeneous spaces are also

Zentralblatt MATH Database 1931 – 2010

© 2010 European Mathematical Society, FIZ Karlsruhe & Springer-Verlag

considered, with basic example $S^n \cong \mathrm{SO}(n+1)/\mathrm{SO}(n)$.

In the whole, including background material and new results about operators on Lie groups, the book offers a pleasant and profitable reading to graduate students and specialists.

Luigi Rodino (Torino)

Keywords : compact Lie groups; operators on the n -torus; toroidal symbols

Classification :

- *35-02 Research monographs (partial differential equations)
- 35S05 General theory of pseudodifferential operators
- 58J40 Pseudodifferential and Fourier integral operators on manifolds
- 43Axx