Imperial College
London

Analysis and Partial Differential Equations

26-30 September 2016

Pure Analysis and PDE Group
Imperial College London, UK
http://wwwf.imperial.ac.uk/ ruzh/IC-conference-oda.htm

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Chiara Taranto
Julio Delgado
Massimiliano Esposito
Michael Ruzhansky
Niyaz Tokmagambetov
Nurgissa Yessirkegenov
Rauan Akylzhanov
Thomas Baker

We kindly acknowledge the support of EPSRC GCRF Knowledge Exchange Scheme and Imperial College London.

1 Abstracts

Generalized fractional differential and integral operators: Properties and some applications

Praveen Agarwal
Anand International College of Engineering, Jaipur, India
goyal.praveen2011@gmail.com

In this talk, we aim to discus some related works on fractional operators. After that we study extended fractional differential operators(such as the Riemann-Liouville and Caputo type fractional operators) involving hypergeometric function introduced by Srivastava et al. and their properties. Moreover, we as well, study certain fractional integral inequalities associated with the well known Pólya-Szegö and Chebyshev type integral inequalities.

$L^p - L^q$ bounds for pseudo-differential operators on locally compact groups

Rauan Akylzhanov
Imperial College London, United Kingdom
r.akylzhanov14@imperial.ac.uk

(Joint work with Michael Ruzhansky) In this talk, we present sufficient conditions [1] for the L^p-L^q boundedness of pseudo-differential operators on locally compact groups. Our approach is based on the operator algebras techniques. The result depends on a version of the Hausdorff-Young-Paley inequality that we establish for this purpose. We shall discuss a recently obtained version of Lizorkin Fourier multiplier theorem on compact Lie groups. As an application, we prove Sobolev embedding inequalities for sub-Laplacians on Lie groups. Following recent development in [2], we briefly sketch extensions of our results to the quantum setting.

References

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On a new integrable PDE for strongly correlated Bose-Einstein condensates

Alexis Arnaudon
Imperial College London, United Kingdom
alexis.arnaudon@imperial.ac.uk

This talk will be a quick journey through a few topics in mathematics and physics. I will start in the physical world of Bose-Einstein condensates (BECs) that can be mathematically understood with PDEs such as the nonlinear Schrödinger equation (NLS). In this well developed topic I will take a sharp turn to a more recent phenomenon that arises if the BECs are strongly correlated. In this case a large number of atoms evaporates and one needs a new equation to describe these atoms. This equation was recently derived by M. Kira and is called the hyperbolic Bloch equation (HBE). After coupling this HBE with the NLS in order to describe the dynamics of the entire system, I will apply some approximations that will produce the so-called hyperbolic reduced Maxwell-Bloch equation (hRMB). From here, I will leave the physics and study the mathematics of this new equation which has the remarkable property to be completely integrable. I will discuss this property and show some explicit solutions that may be observable in strongly correlated BECs. This is a joint work with John D. Gibbon, and is available on arXiv with number 1609.00690.

Laplacian operators on solvable Bianchi groups

Zhirayr Avetisyan
University College London, United Kingdom
z.avetisyan@ucl.ac.uk

On a general Lie group that is neither semisimple nor unimodular, the Laplace-Beltrami operator associated to a left invariant Riemannian metric is arguably the only natural second order partial differential operator that is both elliptic and self-adjoint. We consider solvable Bianchi groups which are 3-dimensional real simply connected almost Abelian Lie groups. On these groups we consider the spectral eigenfunction expansions of Laplace-Beltrami operators associated to arbitrary left-invariant Riemannian metrics, and relate them with the unitary representation theory of the groups.

Using alternatives to the Fourier Transform to define Pseudo Differential Operator classes

Tom Baker
Imperial College London, United Kingdom
t.baker15@imperial.ac.uk

(Joint work with Michael Ruzhansky) In this talk we explore the use of the Short Time Fourier Transform and frames to define a class of operators similar to Pseudo-Differential Functions. In building the theory behind them we find new expressions for the expansions of ordinary operators, which may be useful in different contexts.

Mapping properties for operator-valued pseudodifferential operators on toroidal Besov spaces

Bienvenido Barraza
Universidad del Norte Barranquilla, Colombia
bbarraza@uninorte.edu.co

In this talk, we consider toroidal pseudodifferential operators with operator-valued symbols and prove continuity properties on the the toroidal Besov spaces $B^s_{pq}(\mathbb{T}^n, E)$, without assumptions on the underlying Banach spaces E. We prove for suitable symbols $a : \mathbb{T}^n \times \mathbb{Z}^n \to L(E)$ of order m, that the mapping $op[a] : B^{s+m}_{pq}(\mathbb{T}^n, E) \to B^s_{pq}(\mathbb{T}^n, E)$ is continuous, where $op[a]$ is the pseudodifferential operator associated to a. This extends Theorem 4.5 (i) in [1] and Theorem 3.17 in [2].

Work joint with Robert Denk, Jairo Hernandez Monzon and Max Nendel.

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Global pseudo-differential operators on Besov spaces defined in compact Lie groups

Duvan Cardona
Universidad de los Andes, Bogota, Colombia
d.cardona@uniandes.edu.co

In this talk we present recent results on the boundedenes of certain classes of pseudo-differential operators defined on compact Lie groups as an application of the matrix-valued quantization. We also show some applications to PDEs as a consecuence of this work.

Pseudodifferential Operators and metrics on the phase space

Julio Delgado
Imperial College London, United Kingdom
j.delgado@imperial.ac.uk

We will present a mini-course on some basic elements of the theory of Pseudo-differential operators and the notion of metric on the phase-space (Weyl-Hörmander Calculus). We will also include some applications to the analysis of PDEs.

Boundedness, Compactness and Nuclearity of Pseudo-Differential Operators on $L^p(\mathbb{S}^1)$ and $L^p(\mathbb{Z}),\ p \geq 1$

Mohammad Bagher Ghaemi
Iran University of Science and Technology, Tehran, Iran
mghaemi@iust.ac.ir

(Joint work with Majid Jamalpour Birgani and Elmira Nabizadeh Morsalfard)

Let $\mathbb{Z}$ be the set of all integers, $\mathbb{S}^1$ be the unit circle centered at the origin and $1 \leq p < \infty$. In this paper we investigate boundedness and compactness of pseudo-differential operators on $L^p(\mathbb{S}^1)$ and $L^p(\mathbb{Z})$ and Conditions for pseudo-differential operators from $L^p(\mathbb{S}^1)$ into $L^p(\mathbb{S}^1)$ and from $L^p(\mathbb{Z})$ into $L^p(\mathbb{Z})$ to be bounded and compact are presented. Also a necessary and sufficient condition for pseudo-differential operators from $L^{p_1}(\mathbb{S}^1)$ into $L^{p_2}(\mathbb{S}^1)$ and from $L^{p_1}(\mathbb{Z})$ into $L^{p_2}(\mathbb{Z})$ to be nuclear are presented for $1 \leq p_1, p_2 < \infty$. In the cases when $p_1 = p_2$, the trace formulas of pseudo-differential operators based on their symbols are given.

Multipliers on compact Lie groups

Veronique Fischer
University of Bath, Bath, UK
v.c.m.fischer@bath.ac.uk

In this talk, I will present conditions on a Fourier multiplier of a compact Lie group which ensure that the corresponding operator is Lp bounded. They imply the well-known case of spectral multiplier in the Laplace-Beltrami operator, thereby showing that the conditions are sharp. Old and new questions on the subject will be discussed.

Determination of the initial condition in parabolic equations

DINH NHO HAO
Hanoi Institute of Mathematics, Vietnam
hao@math.ac.vn

To evaluate the solution to an initial boundary value problem for parabolic equations we need to know the initial condition which is unfortunately not always given in practice. Our aim is to reconstruct this condition from 1) the observation of the solution at the final time moment, 2) the interior observations, 3) boundary observations. We discuss the ill-posedness of the problem and propose a variational method for solving it. The problem is discretized by the splitting finite difference method and implemented on computer.

Generation of semigroups for operator-valued Fourier multipliers on the torus

JAIRO HERNANDEZ MONZON
Universidad del Norte Barranquilla, Colombia
jahernan@uninorte.edu.co

In this talk it will be considered toroidal Fourier multipliers with operator-valued symbols, their mapping properties and the generation of analytic semigroups on vector-valued Besov and Sobolev spaces. We show that a parabolic toroidal Fourier multiplier generates an analytic semigroup on the Besov space $B^s_{pq}(\mathbb{T}^n, E)$ and on the Sobolev space $W^k_p(\mathbb{T}^n, E)$, where E is an arbitrary Banach space, $1 \leq p, q \leq 1, s \in \mathbb{R}$ and $k \in \mathbb{N}_0$. For the proof of the Sobolev space result, we establish a uniform estimate on the kernel which is given as an infinite parameter-dependent sum. An application to abstract non-autonomous periodic pseudodifferential Cauchy problems gives the existence and uniqueness of classical solutions for such problems.

Work joint with Robert Denk, Jairo Hernandez Monzon and Max Nendel.

Criterion of boundary integral operator

TYNYSBEK KAL'MENOV
Institute of Mathematics and Mathematical Modeling, Kazakhstan
kalmenov@math.kz

Boundary conditions for the volume potential

$$u = \varepsilon_n * f \equiv \int_\Omega \varepsilon_n(x - \xi) f(\xi) d\xi, \tag{1.1}$$

have been found for the first time in the work of T.Sh. Kal'menov and D. Suragan [1]. Here $\varepsilon_n(x)$ is the fundamental solution of the Laplace equation: $-\triangle\varepsilon_n(x) = \delta(x),\ x \in \mathbb{R}^n$.

Firstly, in [1], it is shown that function $u(x)$ independently from f satisfies the boundary condition

$$-\frac{u(x)}{2} + \int_{\partial\Omega} \left\{ \frac{\partial\varepsilon_n}{\partial n_\xi}(x-\xi)u(\xi) - \varepsilon_n(x-\xi)\frac{\partial u}{\partial n_\xi}(\xi) \right\} d\xi = 0,\ x \in \partial\Omega. \quad (1.2)$$

Secondly, the converse is also true. If the function $u \in W_2^2(\Omega)$ satisfies the Poisson equation

$$-\triangle u(x) = f(x),\ \ x \in \Omega,$$

and the boundary condition (1.2), then it is defined by the volume (Newton) potential (1.1).

In [2–4], the authors have found the boundary conditions for a polyharmonic potential for Riesz potential for integrated transport operator, and other classical operators. A similar result is obtained for the thermal potentials [5]. In all these examples, the integral operators are solutions of the corresponding differential equations. These integral operators are uniquely defined by boundary conditions.

Our problem is to find conditions for the kernel $K(x;\xi)$ of the integral operator

$$u = \int_\Omega K(x,\xi)f(\xi)d\xi,$$

under which the integral operator satisfies the correct boundary conditions for the corresponding differential equation.

Throughout this presentation we mainly use techniques from our work [6].

Acknowledgement. Research supported by the grant 4075/GF4 of the Ministry of Education and Science of Republic of Kazakhstan.

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Resolvents of well-posed polyharmonic operator

Baltabek Kanguzhin
Al–Farabi Kazakh National University, Kazakhstan
kanbalta@mail.ru

Here we describe a class of well-posed problems for the polyharmonic operator in a punctured domain. Some formula is proven for the resolvents under finite-rank perturbations.

Sharpness of the normality condition to global hypoellipticity on closed manifolds

Alexandre Kirilov
Federal University of Paraná - UFPR, Brazil
alexandrekirilov@gmail.com

In this work we investigate the Global Hypoellipticity (GH) of the following class of operators

$$L \doteq D_t + C(t,x,D_x), (t,x) \in \mathbb{T} \times M \tag{1.3}$$

where $\mathbb{T} = \mathbb{R}/2\pi\mathbb{Z}$ stands for the flat torus, M is a n-dimensional closed smooth manifold endowed with a positive measure dx, and $C(t,x,D_x)$ is a pseudo-differential operator on M smoothly depending on the periodic variable t.

Our crucial hypothesis is inspired by the paper of Greenfield-Wallach (Remarks on global hypoellipticity, *Trans. AMS*, 1973), where the authors investigate the (GH) of invariant differential operators with respect of the eigenspaces of a fixed elliptic normal differential operator E as well as the recent generalization of the notion of invariance for elliptic pseudo-differential operators on compact manifolds by Delgado-Ruzhansky (Fourier multipliers, symbols and nuclearity on compact manifolds, arXiv:1404.6479v2, 2015).

We fix an elliptic pseudo-differential operator $E(x,D_x)$ on M, and assume the following conditions:

1. $[C(t,x,D_x), E(x,D_x)] = 0, \quad \forall t \in \mathbb{T};$
2. $C(t,x,D_x)$ is normal, namely, $C^*C = C\,C^*$.

The purpose of this presentation is to investigate the invariance of the (GH) of the operator L without requiring the condition of Greenfield-Wallach of $C(t,x,D_x)$ to be normal.

The idea is to consider small perturbations of the operator L with E-invariant operators which are not normal

$$L^\varepsilon u = D_t u + C^\varepsilon(t,x,D_x)u = f, \quad (t,x) \in \mathbb{T} \times M.$$

We show that the (GH) for such L^ε, arbitrarily close to normal operators, might fail.

This is a joint work with F. de Ávila, UFPR.

Generalized complex fractional Zener model for the wave equation

Sanja Konjik
University of Novi Sad, Serbia
sanja.konjik@dmi.uns.ac.rs

Fractional differential and integral calculus has become an indispensable tool for research and modelling in various scientific fields such as physics, mechanics, engineering, economy, finance, biology, medicine, etc. In particular, linear viscoelasticity benefits a lot from the use of derivatives of noninteger order for describing viscoelastic properties of materials. So far mostly fractional derivatives of real order have been used for that purpose. Applications of fractional differential operators of complex order can be found in the literature, however questions concerning well-posedness of the corresponding models, i.e., restrictions on parameters in constitutive equations that follow from the Second Law of Thermodynamics, were not discussed.

The purpose of this work is to provide a careful investigation of the models of waves in viscoelastic media described by the use of complex order fractional derivatives. We shall present some recent results on physical and mathematical constraints for constitutive equations with derivatives of complex order. The main discussion will be about our current work on generalized complex fractional Zener model for the wave equation.

This talk is based on joint work with Teodor M. Atanacković, Marko Janev and Stevan Pilipović from the University of Novi Sad, Serbia.

Spectral properties of the fractional Laplacian on intervals

Pedro Mendez Hernandez
Universidad de Costa Rica, Costa Rica
pedro.mendez@ucr.ac.cr

We review some recent results on eigenvalues and eigenfunctions of the Dirichlet fractional Laplacian on interval. We discuss, in particular, the behavior of the ground state eigenfunction and bounds on the spectral gap.

Analytical and numerical studies on solitons of a system with two coupled Schrödinger equations with time-modulated nonlinearities

Juan Carlos Munoz
Universidad del Valle, Colombia

jcarlmz@yahoo.com

We show the existence of waveforms of finite energy (vector solitons) for a coupled non-linear Schrödinger system with inhomogeneous coefficients, by employing the theory of positive operators in a cone in a Fréchet space. Furthermore, some of these solutions are approximated by a Newton-type iteration, combined with a collocation-spectral strategy to discretize the corresponding soliton equations. Some numerical simulations concerned with analysis of a collision of two oncoming vector solitons of the system are also performed.

Juan Carlos Muoz was supported by the Department of Mathematics at Universidad del Valle (Colombia) and Colciencias under the research project 1106-712-50006.

Interpolation theorem for Morrey-type spaces and its corollaries

Erlan Nursultanov
Moscow State University, Kazakh Branch, Astana, Kazakhstan
er-nurs@yandex.ru

Let $G=\{G_t\}_{t>0}$ be a family of μ- measurable sets of positive measure such that

$$G_t\subset G_s \quad \text{if} \;\; t\le s.$$

Let $0<p,q\le\infty,\;\; 0<\alpha<\infty,\;\; G(x)=\{G_t+x\}_{t>0}$. Define spaces $LM^{\alpha}_{p,q}(G(x),\mu)$, $M^{\alpha}_{p,q}(G,\mu)$

$$LM^{\alpha}_{p,q}(G(x),\mu)=\left\{f:\;\left(\int_0^\infty\left(t^{-\alpha}\|f\|_{L_p(G_t+x,\mu)}\right)^q\frac{dt}{t}\right)^{1/q}<\infty\right\},$$

$$M^{\alpha}_{p,q}(G(x),\mu)=\left\{f:\;\left(\int_0^\infty\left(t^{-\alpha}\sup_x\|f\|_{L_p(G_t+x,\mu)}\right)^q\frac{dt}{t}\right)^{1/q}<\infty\right\}$$

If $G_t=B_t(0)$ is the ball with center 0 and radius t then $LM^{\alpha}_{p,q}(G,\mu)=LM^{\alpha}_{p,q,z}$ is the local Morrey space and $M^{\alpha}_{p,\infty}(G,\mu)=M^{\alpha}_p$ is the Morrey space.

Theorem Let $0<\alpha_0<\alpha_1<\frac{n}{p},\;\; 0<\beta_0<\beta_1<\frac{n}{q},\;\; 0<p,q<\infty,\;\; \theta\in(0,1)$, $\alpha=(1-\theta)\alpha_0+\theta\alpha_1,\; \beta=(1-\theta)\beta_0+\theta\beta_1$. Let $f\in M^{\alpha}_{p,\tau}(G)$ and A is quasi-linear operator:

$$\|Af\|_{LM^{\beta_0}_{q,\infty}(G(x))}\le M_0\|f\|_{LM^{\alpha_0}_{p,1}(G(x))},\qquad x\in R^n,\;\; f\in LM^{\alpha_0}_{p,1,x}$$

$$\|Af\|_{LM^{\beta_1}_{q,\infty,x}(G(x))}\le M_1\|f\|_{LM^{\alpha_1}_{p,1,x}(G(x))}\qquad x\in R^n\;\; f\in LM^{\alpha_1}_{p,1,x}.$$

Then Af belongs to the space $M^{\beta}_{q,\tau}$ and we have the following inequality

$$\|Af\|_{M^{\beta}_{q,\tau}(G)}\le cM_0^{1-\theta}M_1^{\theta}\|f\|_{M^{\alpha}_{p,\tau}(G)}.$$

This theorem is then applied to obtaining the boundedness in the introduced Morrey-type spaces of the Riesz potential and singular integral operator

The author was supported by MES RK grant 4080/GF-4.

Solvability and microlocal analysis of Eringen's fractional wave equation

Ljubica Oparnica
University of Novi Sad, Serbia
ljubica.oparnica@gmail.com

Eringen's fractional wave equation models elastic wave dispersion in small scale structures as micro and nanostructures. We derive such equation using generalized Eringen's constitutive equation (instead Hook's law) in the system describing waves. For obtained equation we establish existence and uniqueness of solution, along with basic regularity properties and for the solution we determine wave front set.

Correct Statement Boundary Value Problems for new class Singular Integral Equation by Tube Domain

Nusrat Rajabov
Tajik National University, Dushanbe, Tajikistan
nusrat38@mail.ru

Let Ω denote the tube domain $\Omega = \{(x,t) : a < t < b, |z| < R\}$. Lower basis of this cylinder will be denoted by $D = \{t = a, |z| < R\}$ and lateral surface will be denoted by $S = \{a < t < b, |z| = R\}, z = x + iy$. In Ω we consider the following integral equation

$$\varphi(t,z) + \int_a^t \frac{A(\tau)\varphi(\tau,z)}{\tau - a} d\tau - \frac{\lambda}{\pi} \int_D \int \frac{\exp[i\theta]\varphi(t,s)}{(R-\rho)^\beta (s-z)} ds$$

$$-\frac{1}{\pi}\int_a^t \frac{d\tau}{\tau - a} \int_D \int \frac{\exp[i\theta] B(t,s)\varphi(\tau,s)ds}{(R-\rho)^\beta (s-z)} = f(t,s), \quad (1)$$

where $A(t), B(t,z), f(t,z)$ are given real functions, D, Ω are domains, $\theta = \arg s, s = \xi + i\eta, ds = d\xi d\eta$, and $\varphi(t,z)$ is unknown function, $\rho^2 = \xi^2 + \eta^2$, $\lambda = const$, $\beta > 1$.

The solution to this equation is sought in the class of function $\varphi(t,z) \in C(\overline{\Omega})$, $\varphi(a,z) = 0$, $\varphi(t, Re^{i\theta}) = 0, \theta = \arg z$, and its asymptotic behavior for $t \to a$ and $R \to r$ is given in the following formulas

$$\varphi(t,z) = o[(t-a)^\varepsilon], \varepsilon > 0$$

at $t \to a$,

$$\varphi(t,z) = O[(R-r)^{(\delta_1)}], \delta_1 > \beta - 1$$

at $r \to R$.

For integral equation (1), a condition to functions is present in kernels is found, at fulfillment of which, problem of finding solution to this type of integral equations is reduced to problem of finding two splitting systems of integral equations. In this case solution considered integral equation is found in explicit form. In this cases, when general solution contain arbitrary function, inverse formula is found, that is arbitrary functions found by valued solution integral equation in surface cylinder. In this case, when functions present in kernels not among themselves, in modeling cases the solution of integral equation is found in form of absolutely and uniformly converging and generalized power series by powers $(t - a)$, by infinite numbers of analytic functions' variable points of lower basis of this cylinder. Inverse formula is found, that is arbitrary analytic functions found by valued solution of integral equation and its derivative at $t = 0$. Integral representation and its inverse formula is obtained, which is used for standing and investigation of correct Dirichlet type boundary value problems.

On Population Dynamics Models with Stochastic Perturbations

Alexandra Rodkina
University of the West Indies, Kingston, Jamaica
alexandra.rodkina@uwimona.edu.jm

Most perturbations which occur in population dynamics are stochastic and not deterministic. We explore chaotic and stable behaviour of the map with stochastic perturbations having nonzero expectations which in population ecology corresponds to the assumption that immigration at a certain level is more probable than emigration and demonstrate that for a wide class of maps there are stable orbits for large enough values of the parameter.

For the models with the Allee effect and sustainable dynamics for a large interval of values we show that introduction of potentially large enough stochastic perturbation can lead to persistence for any initial values. For smaller perturbation amplitude there are three types of initial values: attracted to low dynamic a.s., a.s. persistent and those which can demonstrate each type of dynamics with a positive probability.

We also discuss stabilisation of difference equation by introducing the stochastic prediction-based control and proportional feedback control, which are used in population dynamics to manage the population size.

Fourier Multipliers on $SU_q(2)$

Carlos Andres Rodriguez
Universidad de los Andes,Bogota, Colombia
ca.rodriguez14@uniandes.edu.co

In this talk we will see the construction of a Fourier Analysis on the quantum group $SU_q(2)$ and then use this to obtain the definition of global Fourier Multipliers.

An isoperimetric inequality for heat potential

Makhmud Sadybekov
Institute of Mathematics and Mathematical Modeling, Kazakhstan
sadybekov@math.kz

Historically, for the first time of scientific literature, in Rayleigh's famous book "The Theory of Sound" (first published in 1877), by using some explicit computation and physical interpretations, he stated that a disk minimizes (among all domains of same area) the first eigenvalue of the Dirichlet Laplacian and the norm of an inverse operator is maximized in a disk among all Euclidean domains of a given area. The proof of this conjecture was obtained after about 50 years later, simultaneously (and independently) by G. Faber and E. Krahn. Nowadays, the inequalities of such type (which are called now the Rayleigh-Faber-Krahn inequality) has been obtained for a wide class of other operators. The corresponding references one can see in [1]-[2].

In this paper the Rayleigh-Faber-Krahn type inequalities for the heat potential operator

$$Hf(x,t) = \int_0^t \int_\Omega K(|x-\xi|, t-\tau) f(\xi,\tau) d\tau d\xi, \quad f \in L^2(D)$$

is proved. Where $\Omega \subset R^d$ be a bounded open set, $D := \Omega \times (0,T)$, and

$$K(x,t) := \frac{\theta(t)}{(2a\sqrt{\pi}t)^{\frac{d}{2}}} e^{-\frac{|x|^2}{4t}}.$$

It is proved that the domain minimizing the first characteristic number of the integral operator H (among all domains of a given measure in R^d) is a ball. Except the mathematical interest to spectral properties this result gives opportunity for obtaining a priori estimates for operator norm of the heat potential on arbitrary cylindrical domains.

This paper is inspired by recent works of Ruzhansky and Suragan [1] and [2] in which analogues of the Rayleigh-Faber-Krahn inequalities for self-adjoint convolution type operators were studied. Unlike their results we consider the integral operator which is Volterra. Therefore this operator does not have eigenvalues. Therefore we obtain Rayleigh-Faber-Krahn inequalities not for the eigenvalues but for characteristic numbers.

Acknowledgement. Research supported by the grant 0824/GF4 of the Ministry of Education and Science of Republic of Kazakhstan.

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Geometric spectral invariants in functorial QFTs

Simon Scott
King's College London, United Kingdom
simon.scott@kcl.ac.uk

TQFT, and other functorial QFTs, provide an impactful viewpoint on certain areas of modern topology. Here, we will contemplate how methods of micro local analysis and, spectral geometric index invariants in the form of generalised Reidemeister torsions, adapt to this category theoretic context.

Well-posedeness of the sub-Laplacian wave equation on stratified Lie groups and Sub-Laplacian Gevrey Spaces

Chiara Taranto
Imperial College London, United Kingdom
c.taranto13@imperial.ac.uk

In a recent work [1], C. Garetto and M. Ruzhansky investigate the Cauchy problem for the time-dependent wave equation for sums of squares of vector fields on compact Lie groups. In particular, they establish the well-posedness in spaces that compare to the Gevrey spaces. In this talk a generalisation of their result to all stratified Lie groups is presented. Furthermore, modelled on the spaces of *Gevrey-type* appearing in [1], we define the *sub-Laplacian Gevrey spaces* on manifolds and partially characterise these spaces. Finally we consider the case of the Heisenberg group, which allows us to give a full characterisation for the *sub-Laplacian Gevrey spaces.*

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Nonharmonic analysis

Niyaz Tokmagambetov
Imperial College London, United Kingdom
Al–Farabi Kazakh National University, Kazakhstan
n.tokmagambetov@imperial.ac.uk

(Joint work with Michael Ruzhansky) In this paper we develop the global symbolic calculus of pseudo–differential operators generated by a boundary value problem for a given (not necessarily self-adjoint or elliptic) differential operator. For this, we also establish elements of a non-self-adjoint distribution theory and the corresponding biorthogonal Fourier analysis. We give applications of the developed analysis to obtain a-priori estimates for solutions of boundary value problems that are elliptic within the constructed calculus.

An adaptive logarithmic threshold framelet analysis of the partial surge initiated instability in a transonic axial flow compressor

Haihui Wang
Beihang University, Beijing, China
whhmath@buaa.edu.cn

Partial surge is a new type of instability inception in transonic axial flow compressors and occurs in the form of axisymmetric low-frequency disturbances localized in the hub region. The frequency of partial surge is about 12.5 Hz at 88developed rotating stall cells in the final phase of the evolution is around 190 Hz. As there are two different frequencies in the instability evolution of partial surge, it is difficult to well denoise the experimental data by general data processing methods that are focused on one single frequency. Considering the unique feature of the partial surge initiated instability, an adaptive logarithmic threshold framelet method has been built. By this method, the well-denoised results can be obtained. Based on the results, the instability evolution of partial surge is clearly shown. At the same time, several new findings about partial surge have been presented: (1) the propagating procedure of one partial surge disturbance: the amplitude of disturbance is increasing until one partial surge disturbance is developed; (2) the occurrence of rotating stall cells is related to the partial surge disturbance; (3) the propagating procedure of one rotating stall cell is presented; (4) the partial surge disturbance is well proved to be axisymmetric. Finally, the capability of this denoise method is also discussed.

Euler-Hilbert-Sobolev and Sobolev-Lorentz-Zygmund spaces on homogeneous groups

Nurgissa Yessirkegenov
Imperial College London, United Kingdom
n.yessirkegenov15@imperial.ac.uk

In this talk, we present Euler-Hilbert-Sobolev spaces and obtain embedding results on homogeneous groups using Euler operators, which are homogeneous differential operators of order zero. Sharp remainder terms of L^p and weighted Sobolev and Sobolev-Rellich inequalities on homogeneous groups are given. As consequences, we obtain analogues of the generalised classical Sobolev and Sobolev-Rellich inequalities. We also discuss applications of logarithmic Hardy inequalities to Sobolev-Lorentz-Zygmund spaces.

I am grateful to Professor M.Ruzhansky and D.Suragan for the many instructive discus-

sions.

2 Additional Abstracts

On a heat transfer model for the locally inhomogeneous initial data

Gaukhar Arepova
Institute of Mathematics and Mathematical Modeling, Kazakhstan
Al-Farabi Kazakh National University, Kazakhstan
arepova1@math.kz

We consider a model case of the problem of heat diffusion in a homogeneous body with a special initial state. The peculiarity of this initial state is its local inhomogeneity. That is, there is a closed domain Ω inside a body, the initial state is constant out of the domain. Mathematical modeling leads to the problem for a homogeneous multi-dimensional diffusion equation.

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with sufficiently smooth boundary $\partial\Omega$. Through D we denote a cylindrical domain $D = \Omega \times (0, T)$. In D we consider a surface heat potential

$$u = \int_{\Omega} \varepsilon_n(x - \xi, t) u_0(\xi) d\xi, \tag{2.1}$$

where

$$\varepsilon_n(x, t) = \theta(t)(2a\sqrt{\pi t})^{-n} e^{-\frac{|x|^2}{4a^2 t}}$$

is a fundamental solution of the heat equation

$$\Diamond u \equiv \left(\frac{\partial}{\partial t} - a^2 \triangle_x\right) u = 0. \tag{2.2}$$

We construct the boundary conditions on the boundary of the domain Ω, which can be characterized as "transparent" boundary conditions . We separately consider a special case – a model of redistribution of heat in a uniform linear rod, the side surface of which is insulated in the absence of (internal and external) sources of heat and of locally inhomogeneous initial state.

Throughout this note we mainly use techniques from works [1–3].

Acknowledgement Research supported by the grant 4075/GF4 of the Ministry of Education and Science of Republic of Kazakhstan.

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Stability of basis property of a type of problems with nonlocal perturbation of boundary conditions

Gulnar Dildabek
Institute of Mathematics and Mathematical Modeling, Kazakhstan
Al-Farabi Kazakh National University, Kazakhstan
dildabek@math.kz

In the present paper we investigate a nonlocal boundary problem for the Laplace equation in a half-disk, with opposite flows at the part of the boundary. Our goal is to find a function $u(r,\theta) \in C^0(\bar{D}) \cap C^2(D)$ satisfying in D the equation

$$\Delta u = 0$$

with the boundary conditions

$$u(1,\theta) = f(\theta),\ 0 \leq \theta \leq \pi,$$

$$u(r,0) = 0, r \in [0,1],$$

$$\frac{\partial u}{\partial \theta}(r,0) = -\frac{\partial u}{\partial \theta}(r,\pi) + \alpha u(r,\pi), \quad r \in (0,1)$$

where $D = \{(r,\theta): \quad 0 < r < 1, 0 < \theta < \pi\}$; $\alpha < 0$; $f(\theta) \in C^2[0,\pi]$, $f(0) = 0$, $f'(0) = -f'(\pi) + \alpha f(\pi)$.

The difference of this problem is the impossibility of direct applying of the Fourier method (separation of variables). Because the corresponding spectral problem for the ordinary differential equation has the system of eigenfunctions not forming a basis. Throughout this note we mainly use techniques from our works [1, 2].

A special system of functions based on these eigenfunctions is constructed. This system has already formed the basis. This new basis is used for solving of the nonlocal boundary value problem. The existence and the uniqueness of the classical solution of the problem are proved.

Acknowledgement Research supported by the grant 0825/GF4 of the Ministry of Education and Science of Republic of Kazakhstan.

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Stability of basis property of a type of problems with nonlocal perturbation of boundary conditions

Nurlan Imanbaev
Institute of Mathematics and Mathematical Modeling, Kazakhstan
South Kazakhstan State Pedagogical Institute, Kazakhstan
imanbaev@math.kz

This report is devoted to a spectral problem for a multiple differentiation operator with an integral perturbation of boundary conditions of one type which are regular, but not strongly regular:

$$l(u) \equiv -u''(x) = \lambda u(x),\ 0 < x < 1, \tag{2.3}$$

$$U_1(u) \equiv u'(0) - u'(1) + \alpha u(0) = \int_0^1 \overline{p(x)} u(x)dx,\ p(x) \in L_2(0,1), \tag{2.4}$$

$$U_2(u) \equiv u(0) - u(1) = 0. \tag{2.5}$$

Here $\alpha \neq 0$ is an arbitrary complex number.

The unperturbed problem ($p(x) \equiv 0$) has an asymptotically simple spectrum, and its system of normalized eigenfunctions creates the Riesz basis. We construct the characteristic determinant of the spectral problem with an integral perturbation of the boundary conditions. The perturbed problem can have any finite number of multiple eigenvalues. Therefore, its root subspaces consist of its eigen and (maybe) adjoint functions. It is shown that the Riesz basis property of a system of eigen and adjoint functions is stable with respect to integral perturbations of the boundary condition.

Throughout this note we mainly use techniques from our works [1–4].

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On an illposed model of oscillations of a flat plate with a variety of mounts on opposite sides

Ulzada Iskakova
Institute of Mathematics and Mathematical Modeling, Kazakhstan
iskakova@math.kz

We consider a model case of stationary vibrations of a thin flat plate, one side of which is embedded, the opposite side is free, and the sides are freely leaned. In mathematical modeling there is a local boundary value problem for the biharmonic equation in a rectangular domain. Boundary conditions are given on all boundary of the domain.

Problem C. *Find in* $D = \{(x,y) : 0 < x < \pi, 0 < y < l\}$ *a solution to the biharmonic equation*

$$\triangle^2 u \equiv u_{xxxx}(x,y) + 2u_{xxyy}(x,y) + u_{yyyy}(x,y) = 0, \quad (x,y) \in D,$$

satisfying boundary conditions in the first spatial variable x*:*

$$u|_{x=0} = 0, \ \triangle u|_{x=0} = 0; \quad u|_{x=\pi} = 0, \ \triangle u|_{x=\pi} = 0; \quad 0 \le y \le l;$$

and boundary conditions in the second spatial variable y*:*

$$u|_{y=0} = \varphi_1(x), \ \frac{\partial u}{\partial y}\Big|_{y=0} = \varphi_2(x), \quad 0 \le x \le \pi;$$

$$\triangle u|_{y=l} = \psi_1(x), \quad \frac{\partial \triangle u}{\partial y}\Big|_{y=l} = \psi_2(x), \ 0 \le x \le \pi.$$

We show that the considered problem is self-adjoint. Herewith the problem is ill-posed. We show that the stability of solution to the problem is disturbed. Necessary and sufficient conditions of existence of the problem solution are found. Spaces of the ill-posedness of the considered problem are constructed.

Throughout this note we mainly use techniques from our works [1-3].

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On spectral properties of Laplace operator in punctured domains

Gulzat Nalzhupbayeva
Imperial College London, United Kingdom
Al–Farabi Kazakh National University, Kazakhstan
n.gulzat@imperial.ac.uk

Here we consider well-posed perturbations of the Laplace operator in punctured domains. Properties of correct perturbations are established. In particular, we describe some identities for eigenvalues of these operators.

Inverse coefficient problems for one-dimensional heat transfer with a preservation of medium temperature condition

Gulaiym Oralsyn
Institute of Mathematics and Mathematical Modeling, Kazakhstan
Al-Farabi Kazakh National University, Kazakhstan
oralsyn@math.kz

We examine mathematical model models describing the process of heat transfer in a homogeneous bar with a prescribed law of changes of the average temperature. So there is an inverse problem for the heat equation in which together with finding the solution of the equation it is required to find unknown coefficient depending only on the time variable.

In the domain $\Omega = \{(x,t) : 0 < x < 1, 0 < t < T\}$ consider a problem on finding unknown coefficient $p(t)$ of the heat equation

$$u_t = u_{xx}(x,t) - p(t)u(x,t) + f(x,t)$$

and its solution satisfying the initial condition

$$u(x,0) = \varphi(x),\ 0 \le x \le 1,$$

the nonlocal boundary condition

$$u_x(0,t) = u_x(1,t) - \alpha u(1,t),\ \ u(0,t) = 0,\ \ 0 \le t \le T,$$

and the overdetermination conditions

$$\int_0^1 u(x,t)dx = E(t),\ E(t) \ne 0, 0 \le t \le T,$$

where $E(t) \in W_2^1(0,T)$. Here the parameter α is any positive number, and $f(x), \varphi(x)$ and $E(t)$ are given functions.

In [1] (for $\alpha < 0$) some of these conditions were removed and it is shown that the inverse problem has the unique generalized solution. This work is a continuation of these studies. We consider the case $\alpha > 0$.

Throughout this note we mainly use techniques from our works [1–2].

Acknowledgement. Research supported by the grant 0824/GF4 of the Ministry of Education and Science of Republic of Kazakhstan.

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Construction of mathematical models of extraction processes with non-local conditions by a spatial variable

Issabek Orazov
Institute of Mathematics and Mathematical Modeling, Kazakhstan
M. Auezov South Kazakhstan State University, Kazakhstan
orazov@math.kz

In this paper we consider issues of constructing mathematical models of extraction processes from solid polydisperse porous materials considering the porosity of structure of particles, taking into account the connection of the residence time of fractions with particle size in the extractant, based on inverse problems of recovery of coefficients of diffusion processes under various variants of boundary conditions by a spatial variable.

We consider a mathematical model which models the extraction process of a target component from the polydispersed porous material. The suggested model is demonstrated by the example of a at solid material with bidispersed pores of different size in the form of a system of channels of macropores with micropores facing their walls. The macropores and the micropores in the material have homogeneous size. We model a case when micropores of the solid material (dispersed medium) are initially filled with an oil (dispersion phase), which is our target component. And the macropores are filled in with a pure solvent. In the process of extraction the oil diffuses from the micropore to the macropore, and then from the micropores to the external solvent volume, wherein the ratio of concentrations in the macropore and the micropore is taken in accordance with the linear law of adsorption. The well-posedness of the formulated mathematical model has been justified.

The theoretical mathematical science has deep enough advanced in solving inverse problems for diffusion processes. And besides, as a rule, the problems are researched under simplest selfadjoint boundary conditions by a spatial variable. Unlike the mentioned works we propose to consider the problems with more general boundary conditions by a spatial variable. The selfadjointness of the boundary conditions is not assumed, only requirement of their regularity by Birkhoff is sufficient. The inverse problems researched by us are directly obtained from mathematical models of technological processes.

Throughout this note we mainly use techniques from our works [1–3].

Acknowledgement Research supported by the grant 0824/GF4 of the Ministry of Education and Science of Republic of Kazakhstan.

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Existence of eigenvalues of problem with shift for an equation of parabolic-hyperbolic type

Aizhan Tengayeva
Institute of Mathematics and Mathematical Modeling, Kazakhstan
Kazakh National Agrarian University, Kazakhstan
tengayeva@math.kz

In the paper a spectral problem for an operator of parabolic-hyperbolic type of I kind with non-classical boundary conditions is considered. The problem is considered in a standard domain. The parabolic part of the space is a rectangle. And the hyperbolic part of the space coincides with a characteristic triangle. We consider a problem with the local boundary condition in the domain of parabolicity and with the boundary condition with displacement in the domain of hyperbolicity.

Let $\Omega \in R^2$ be a finite domain bounded for $y > 0$ by the segments AA_0, A_0B_0, B_0B, $A = (0,0)$, $A_0 = (0,1)$, $B_0 = (1,1)$, $B = (1,0)$, and for $y < 0$ by the characteristics $AC:\ x + y = 0$ and $BC:\ x - y = 1$ of an equation of the mixed parabolic-hyperbolic type

$$Lu = \left\{ \begin{array}{c} u_x - u_{yy}, y > 0 \\ u_{xx} - u_{yy}, y < 0 \end{array} \right\} = f(x,y). \tag{2.6}$$

Problem S. *Find a solution to Eq.* (2.6) *satisfying boundary conditions*

$$u|_{AA_0 \cup A_0 B_0} = 0, \tag{2.7}$$

$$\alpha u\left(\theta_0\left(t\right)\right) = \beta u\left(\theta_1\left(t\right)\right), \quad 0 \le t \le 1, \tag{2.8}$$

where $\theta_0\left(t\right) = \left(\frac{t}{2}, -\frac{t}{2}\right)$, $\theta_1\left(t\right) = \left(\frac{t+1}{2}, \frac{t-1}{2}\right)$.

We prove the strong solvability of the considered problem. The main aim of the paper is the research of spectral properties of the problem. The existence of eigenvalues of the problem is proved.

Throughout this note we mainly use techniques from our work [1].

Acknowledgement. Research supported by the grant 0825/GF4 of the Ministry of Education and Science of Republic of Kazakhstan.

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Inequalities of Hardy-Littlewood Type in Anisotropic Space

Nazerke Tleukhanova
L.N. Gumilyov Eurasian National University, Astana, Kazakhstan
tleukhanova@rambler.ru

(Joint work with Erlan Nursultanov)

Let $1 < p < \infty$ and $\Phi = \{\phi\}_{k=1}^{\infty}$ be the orthonormal system bounded in total, $f \in L_p[0,1]$, $f \overset{\text{a.e.}}{=} \sum_{k=1}^{\infty} a_k \phi_k(x)$. In a case of $1 < p < 2$ the task about properties of summability of Fourier coefficients $\{a_k\}_{k=0}^{\infty}$ can be solved by classical Hardy-Littlewood-Polya inequality [1]

$$\sum_{k=1}^{\infty} k^{p-2} |a_k|^p \le c \|f\|_{L_p}^p. \tag{2.9}$$

For the case when orthonormal system $\Phi = \{\phi\}_{k=1}^{\infty}$ is regular and $2 \le p < \infty$, according to inequality, proved by E.D. Nursultanov ([2]), it follows

$$\sum_{k=1}^{\infty} k^{p-2} |\bar{a_k}|^p \le c \|f\|_{L_p}^p, \tag{2.10}$$

here $\bar{a_k} = \frac{1}{k} \left| \sum_{m=1}^{k} a_m \right|$.

These inequalities are also valid in the multidimensional case (see e.g. [3]). For function from anisotropic spaces with a vector parameter $\mathbf{p} = (p_1, ..., p_n)$, it is necessary to consider a problem of summability of Fourier coefficients when some parameters p_i more than 2, and others less or equal to 2.This article is devoted to the problem.

E. Nursultanov was supported by MES RK grant 4080/GF-4, N. Tleukhanova was supported by MES RK grants 5540/GF-4, 3311/GF-4.

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On a nonlinear problem for the fractional Falkner-Skan type equation

Berikbol T. Torebek
Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan
torebek@math.kz

In this work we consider the nonlinear problem for fractional Falkner-Skan type equation:

$$(\mathcal{D}^{\alpha}u(t))' + \mathcal{M}\left(t, u(t)\right)\mathcal{D}^{\alpha}u(t), a < t < b, \qquad (1)$$

with conditions

$$u(a) = \mathcal{A},\ u(b) = \mathcal{B},\ u'(a) = \mathcal{C}, \qquad (2).$$

Here $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ are given real numbers and $\mathcal{D}^{\alpha}$ is a differentiation operator of the $1 < \alpha < 2$ order in the sense of Caputo [1].

Note that when $\alpha \to 1$ the problem (1), (2) has application in problems of boundary layer theory in fluid mechanics and polymer theory (see. [2]-[4]).

Condition * Let $M\left(t, u(t)\right)$ defined and continuous in a domain

$$\Omega = \{u(t), t : \ \|u\| \le r,\ r > 0,\ a \le t \le b\}$$

and

$$m = \min_{u,t\in\Omega} \mathcal{M}\left(t, u(t)\right),\ M = \max\left\{\max_{u,t\in\Omega} \mathcal{M}\left(t, u(t)\right), 0\right\},$$

where $r = |\mathcal{A}| + (b-a)\,|\mathcal{C}| + |\mathcal{B} - \mathcal{A} - (b-a)\mathcal{C}|$.

Theorem. *Let $M\left(t, u(t)\right)$ satisfies the condition *. Then the problem (1), (2) has a unique solution in $C^3[a,b]$.*

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3 Partipants

Praveen Agarwal, Anand International College of Engineering, Jaipur, India

Rauan Akylzhanov, Imperial College London, United Kingdom

Alexis Arnaudon, Imperial College London, United Kingdom

Zhirayr Avetisyan, University College London, United Kingdom

Tom Baker, Imperial College London, United Kingdom

Bienvenido Barraza, Universidad del Norte Barranquilla, Colombia

Duvan Cardona, Universidad de los Andes, Bogota, Colombia

Julio Delgado, Imperial College London, United Kingdom

Mohammad Bagher Ghaemi, Iran University of Science and Technology, Tehran, Iran

Veronique Fischer, University of Bath, Bath, UK

Dinh Nho Hao, Hanoi Institute of Mathematics, Vietnam

Jairo Hernandez Monzon, Universidad del Norte Barranquilla, Colombia

Tynysbek Kal'menov, Institute of Mathematics and Mathematical Modeling, Kazakhstan

Baltabek Kanguzhin, Al–Farabi Kazakh National University, Kazakhstan

Alexandre Kirilov, Federal University of Paraná - UFPR, Brazil

Sanja Konjik, University of Novi Sad, Serbia

Pedro Mendez Hernandez, Universidad de Costa Rica, Costa Rica

Juan Carlos Munoz, Universidad del Valle, Colombia

Erlan Nursultanov, Moscow State University, Kazakh Branch, Astana, Kazakhstan

Ljubica Oparnica, University of Novi Sad, Serbia

Nusrat Rajabov, Tajik National University, Dushanbe, Tajikistan

Alexandra Rodkina, University of the West Indies, Kingston, Jamaica

Carlos Andres Rodriguez, Universidad de los Andes,Bogota, Colombia

Makhmud Sadybekov, Institute of Mathematics and Mathematical Modeling, Kazakhstan

Simon Scott, King's College London, United Kingdom

Chiara Taranto, Imperial College London, United Kingdom

Niyaz Tokmagambetov, Imperial and Kazakh National University

Haihui Wang, Beihang University, Beijing, China

Nurgissa Yessirkegenov, Imperial College London, United Kingdom

Gulzat Nalzhupbayeva, Imperial and Kazakh National University

Nazerke Tleukhanova, L.N. Gumilyov Eurasian National University, Kazakhstan