LONDON MATHEMATICAL SOCIETY
EST. 1865

UK Network on Hyperbolic Equations and Related Topics, 2018-2019

Supported by LMS (Scheme 3) and by the Edinburgh Mathematical Society

Departments

School of Mathematics, The University of Edinburgh, Maxwell Institute for Mathematical Sciences

Department of Mathematical Sciences, Loughborough University

School of Mathematical Sciences, Queen Mary University London

Organisers

Pieter Blue (Edinburgh)
Claudia Garetto (Loughborough)
Michael Ruzhansky (Queen Mary London)

Linear and nonlinear hyperbolic partial differential equations (PDEs) arise in basically all sciences (physics, chemistry, medicine, engineering, astronomy, etc.). In physics, they model several important phenomena, from propagation of waves in a medium (for instance propagation of seismic waves during an earthquake) to refraction in crystals and gas-dynamics. The purpose of this UK network on hyperbolic equations and related topics is to bring together the expertise on hyperbolic equations of three different mathematics department (Edinburgh, Imperial, Loughborough), to strengthen the existing research collaborations and to create new ones. Three 1-day workshops per year are planned focused on different approaches to hyperbolic equations and related topics (inverse problems, kinetic theory, imaging, microlocal analysis, general relativity, etc.).

Meeting 2

04 March 2019 (Monday), Loughborough University, Room G007 and Room WAV057

Planned program

Room G007

11:00-12:00: **Arick Shao** (QMUL)
On Controllability of Waves and Geometric Carleman Estimates

12:00-13:00 **Jean-Claude Cuenin** (LMU Münich)
Uniform resolvent estimates for the Laplace-Beltrami operator

LUNCH

Room WAV057

14:00-15:00: **Natalia Janson** (Loughborough)
Brain as a dynamical system with plastic self-organising velocity field

15:00-15:30: **Jinrong Bao** (Loughborough)
Separatrix crossing in rotation of a body with changing geometry of masses

15:30-16:00: **Matteo Capoferri** (UCL)
Hyperbolic propagators: from scalar equations to first order systems

COFFEE and NETWORKING

Abstracts

Arick Shao (QMUL)

On Controllability of Waves and Geometric Carleman Estimates

In this talk, we consider the question of exact (boundary) controllability of wave equations: whether one can steer their solutions from any initial state to any final state using appropriate boundary data. In particular, we discuss new and fully general results for linear wave equations on time-dependent domains with moving boundaries. We also discuss the novel geometric Carleman estimates that are the main tools for proving these controllability results.

Jean-Claude Cuenin (LMU Münich)

Uniform resolvent estimates for the Laplace-Beltrami operator

L^p to L^p' resolvent estimates that are uniform in the spectral parameter have become an important tool in many branches of PDE and spectral theory, such as unique continuation, inverse problems or Lieb-Thirring inequalities. In this talk I will give a review of recent developments in the subject, discuss some open problems and make the connection to hyperbolic equations.

Natalia Janson (Loughborough)

Brain as a dynamical system with plastic self-organising velocity field

One of the greatest challenges of modern science is to understand how the brain creates the mind, which should help create human-like artificial intelligence. The dominating paradigm asserts that in order to be intelligent, the system needs to be made as a neural network. Mathematics is used primarily as a tool to model either the brain as a device, or the phenomena occurring in the brain. With this, there is no definitive conceptual model of the brain.

In [1] we look at the brain from the viewpoint of dynamical systems theory and shift the focus from the behaviour to the reasons behind the behaviour. It is well appreciated that the brain can mathematically be described as a dynamical system. Then the force controlling every neuron is the velocity vector field of this model. As with all dynamical models of real devices, this field is a mathematical image of the physical architecture of the brain. The unique feature of the brain, which enables learning, is the self-organised plasticity of its architecture, which implies that its velocity field de-facto evolves with time with account of sensory stimuli. We hypothesize that the brain's velocity field in fact self-organises according to some deterministic rules. Dynamical systems with self-organising velocity fields have not been considered previously, so we introduce them in order to describe cognition. We suggest that the condition for being intelligent is the ability to create a self-organising velocity field evolving according to some appropriate laws while incorporating sensory information. With this, the plastic architecture of the brain serves as the means to create the velocity field with the required properties. To support this hypothesis, we construct a simple non-neuromorphic dynamical system with a self-organising velocity field and demonstrate how it performs basic cognition in line with what is expected of artificial neural networks.

The mathematical consistency of the newly introduced dynamical systems is studied in [2], where the existence and uniqueness of solutions is proved under certain conditions, together with the existence of pullback attractors and forward limit sets.

In the context of neurophysiology, the velocity field represents an explicit link between the brain substance and bodily behaviour. It could help combine a variety of conflicting notions about memory representation and acquisition into a single picture. Generally, revealing the evolution laws of the brain's velocity field could help reveal the laws of human cognition. At the same time, a variety of such laws can be constructed artificially, which could lead to novel types of Artificial Intelligence.

[1] Janson, N.B. & Marsden, C.J. Dynamical system with plastic self-organized velocity field as an alternative conceptual model of a cognitive system. Scientific Reports 7, 17007 (2017). https://www.nature.com/articles/s41598-017-16994-y

[2] Janson, N.B. & Kloeden, P.E. Asymptotic behaviour of dynamical systems with plastic self-organising vector fields, arXiv:1812.05205

(2018)

Jinrong Bao (Loughborough)
Separatrix crossing in rotation of a body with changing geometry of masses

We consider free rotation of a body whose parts move slowly with respect to each other under the action of internal forces. This problem can be considered as a perturbation of the Euler-Poinsot problem. The dynamics has an approximate conservation law - an adiabatic invariant. This allows to describe the evolution of rotation in the adiabatic approximation. The evolution leads to an overturn in the rotation of the body: the vector of angular velocity crosses the separatrix of the Euler-Poinsot problem. This crossing leads to a quasi-random scattering in body's dynamics. We obtain formulas for probabilities of capture into different domains in the phase space at separatrix crossings.

Matteo Capoferri (UCL)
Hyperbolic propagators: from scalar equations to first order systems

In my talk I will present a global, invariant and explicit construction of hyperbolic propagators on closed Riemannian manifolds. This can be achieved by representing the propagator as a single Fourier integral operator with distinguished complex-valued phase function. The knowledge of the propagator allows one, in turn, to recover asymptotic spectral properties of the operators at hand. Proceeding by examples, I will point out similarities and fundamental differences between scalar equations and first order systems.
The talk is based both on recent results (joint with D. Vassiliev and M. Levitin) and on work in progress.