Imperial College London

Aalto-yliopisto

Book "Pseudo-Differential Operators and Symmetries"

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Ghent Analysis PDE

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Description

This monograph is devoted to the development of the theory of pseudo-differential operators on spaces with symmetries. Such spaces are the Euclidean space $\mathbb{R}^n$, the n torus $\mathbb{T}^n$, compact Lie groups and compact homogeneous spaces. The book consists of several parts. One of our aims has been not only to present new results on pseudo-differential operators but also to show parallels between different approaches to pseudo-differential operators on different spaces. Moreover, we tried to present the material in a self-contained way to make it accessible for readers approaching the material for the first time.

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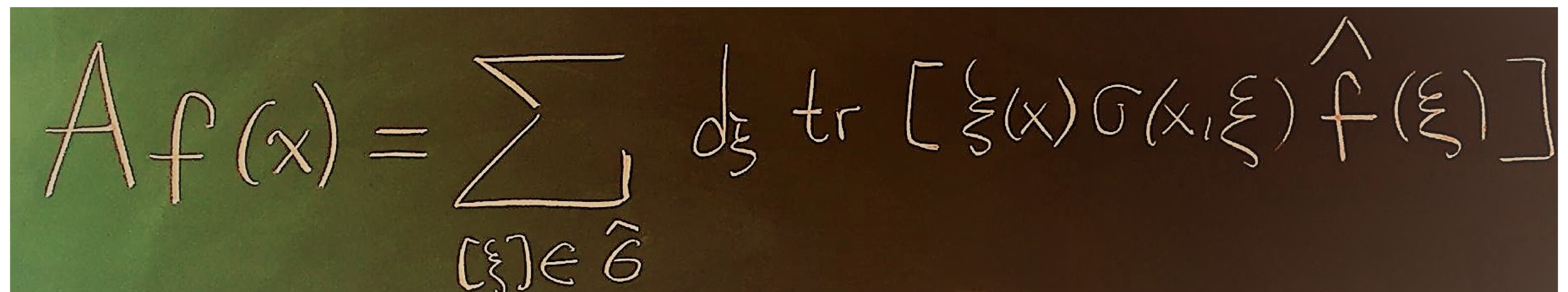
Quantization of operators

Let σ_A be the symbol of a continuous linear operator $A : C^\infty(G) \to C^\infty(G)$. Then

$$Af(x) = \sum_{[\xi]\in\widehat{G}} d_\xi \operatorname{Tr}\left(\xi(x)\sigma_A(x,\xi)\widehat{f}(\xi)\right),$$

for every $f \in C^\infty(G)$ and $x \in G$. Conversely, we have

$$\sigma_A(x,\xi) = \xi(x)^* A\xi(x) \in \mathbb{C}^{d_\xi \times d_\xi}.$$



Compact Lie Groups

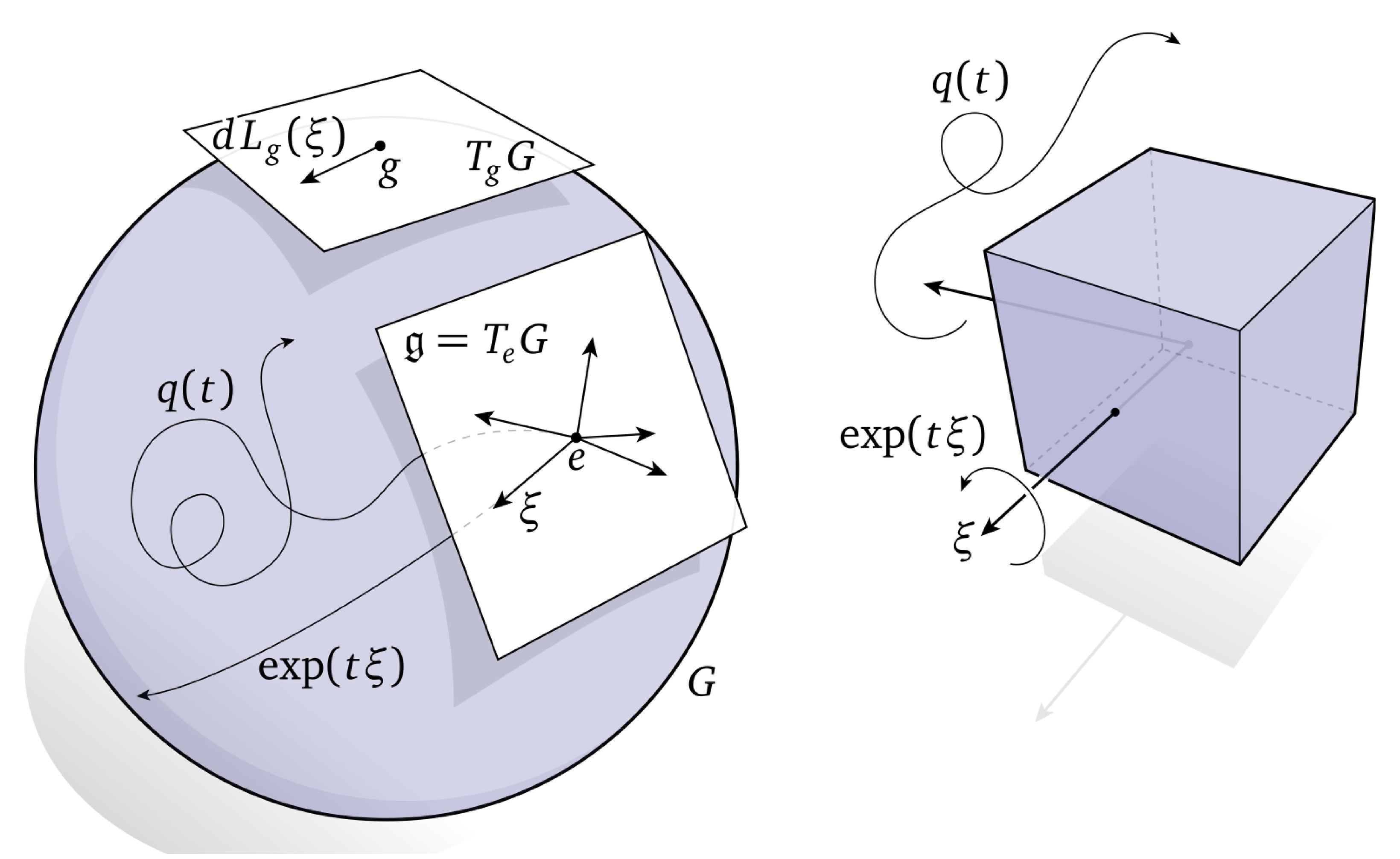


Illustration from keenan.is

Fourier Series on compact Lie groups

If $\xi : G \to U(d_\xi)$ is a unitary matrix representation of a compact Lie group G, then

$$\widehat{f}(\xi) = \int_G f(x)\xi(x)^* dx \in \mathbb{C}^{d_\xi \times d_\xi}$$

has matrix elements

$$\widehat{f}(\xi)_{mn} = \int_G f(x)\overline{\xi(x)_{nm}} dx \in \mathbb{C},\ 1 \le m,n \le d_\xi.$$

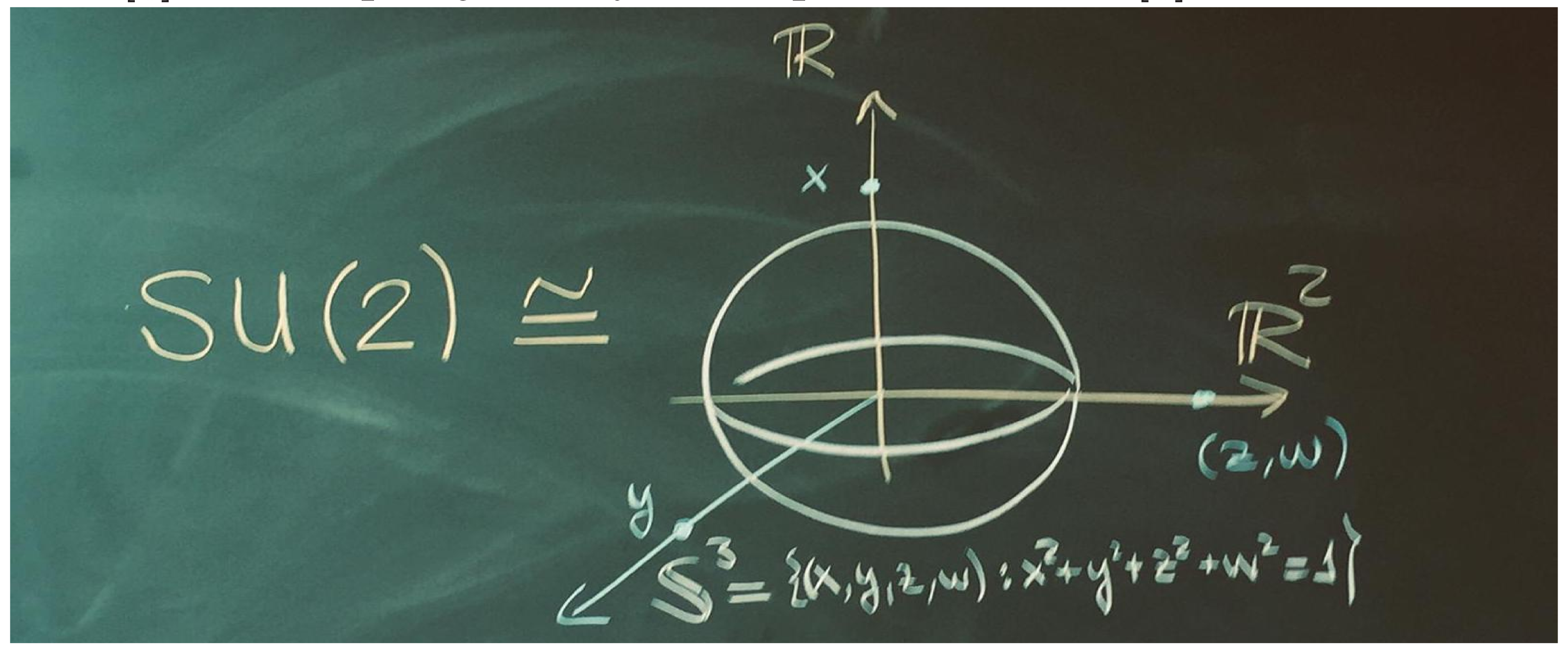
If $f \in L^2(G)$ then

$$\widehat{f}(\xi)_{mn} = (f, \xi_{nm})_{L^2(G)},$$

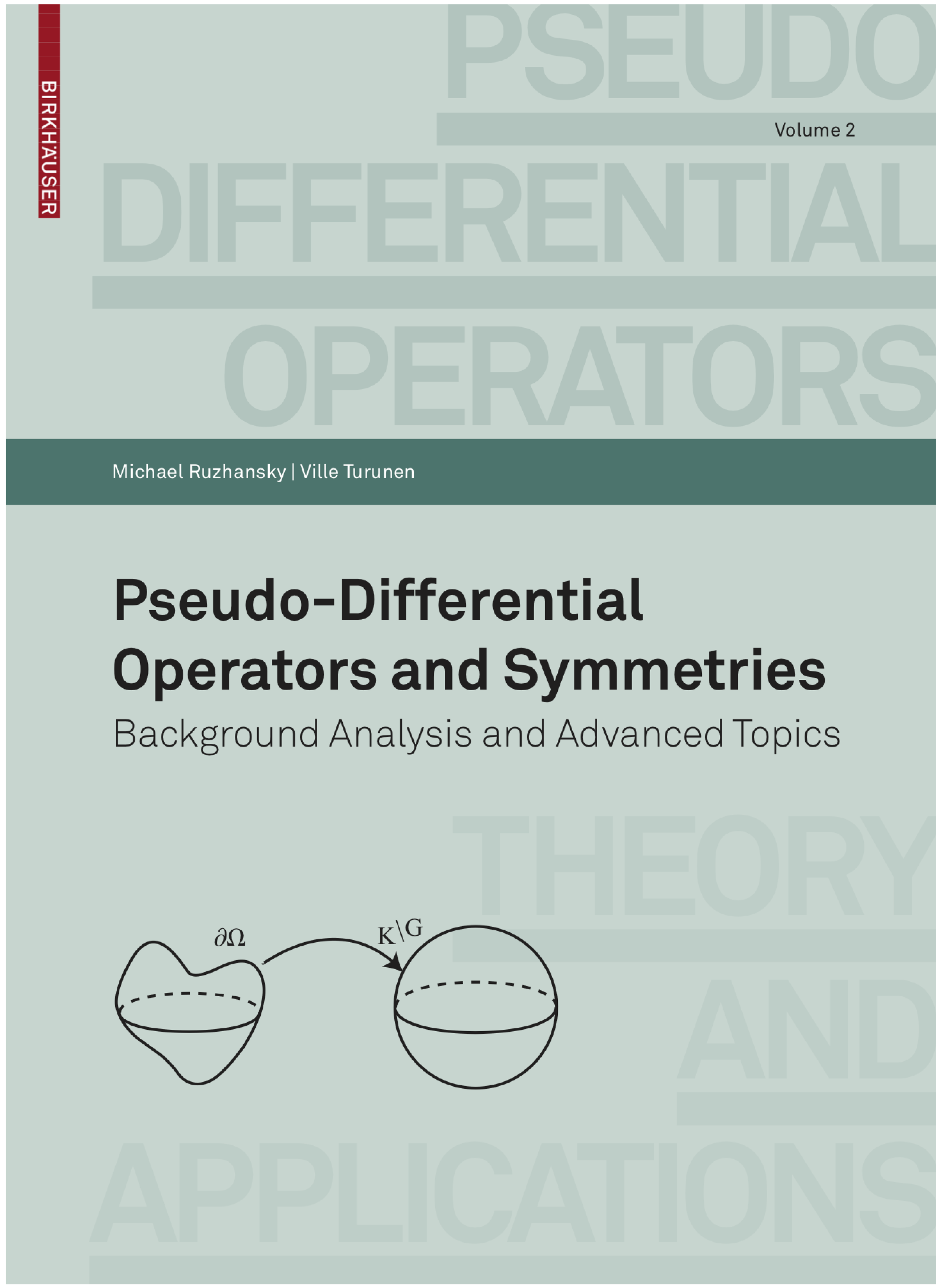
and by the Peter–Weyl Theorem we have

$$f(x) = \sum_{[\xi]\in\widehat{G}} d_\xi \operatorname{Tr}\left(\xi(x)\widehat{f}(\xi)\right),$$

for almost every $x \in G$, where the summation is understood so that for each class $[\xi] \in \widehat{G}$ we pick just (any) one representative $\xi \in [\xi]$.



Book Cover



Information

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